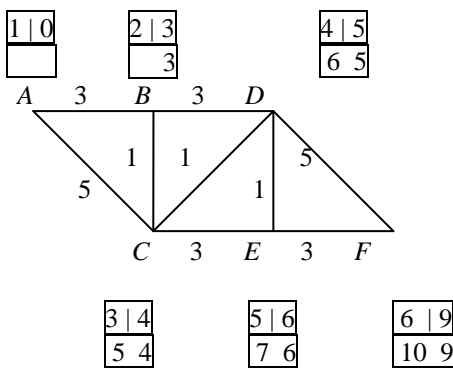


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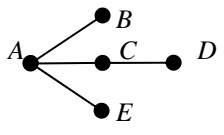
4736 Decision Mathematics 1

TO BE ANSWERED ON INSERT				
1	(i)	 <p>Path: A – B – C – D – E – F Weight: 9</p>	<p>M1 A1</p> <p>Evidence of updating at C, D, E or F All temporary labels correct, with no extras</p> <p>B1</p> <p>All permanent labels correct</p> <p>B1 B1</p> <p>cao cao</p>	[5]
	(ii)	<p>Total weight of all arcs = 25</p> <p>Only odd nodes are B and E. Least weight path joining B to E is B – C – E = 3.</p> <p>Weight: 28</p> <p>Route: (example) A – B – D – F – E – C – B – C – D – E – D – C – A</p>	<p>B1</p> <p>Total weight = 25 (may be implied from weight)</p> <p>M1</p> <p>B to E = 3</p> <p>A1</p> <p>28 (cao)</p> <p>B1</p> <p>A valid closed route that uses BC, CD and DE twice and all other arcs once</p>	[4]
	(iii)	<p>A – B – E – F</p> <p>Graph is now Eulerian, so no need to repeat arcs</p>	<p>B1</p> <p>cao</p> <p>B1</p> <p>Eulerian (or equivalent)</p>	[2]
Total =				11

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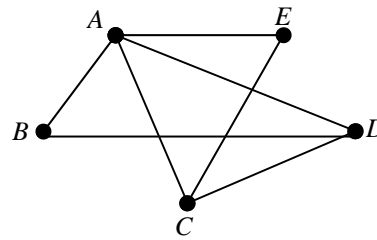
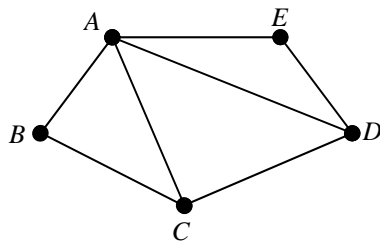
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2	(i)	A graph cannot have an odd number of odd vertices (nodes)	B1	Or equivalent (eg $3 \times 5 = 15 \Rightarrow 7\frac{1}{2}$ arcs) Not from a diagram of a specific case	[1]
	(ii)	It has exactly two odd nodes eg $C A B C D E A D$	B1 B1	2 odd nodes A valid semi-Eulerian trail	[2]
	(iii)	$AE = 2$ $AC = 3$ $AB = 5$ $CD = 7$ Weight = 17 	B1 B1 B1	Correct tree (vertices must be labelled) Order of choosing arcs in a valid application of Prim, starting at A (working shown on a network or matrix) 17	[3]
	(iv)	Lower bound = 29 $A - E - D - F - C - B - A$ $= 34$ $F - C - A - E - D$ and $F - D - C - A - E$ Vertex B is missed out	B1 M1 A1 B1	29 or 12 + their tree weight from (iii) $A - E - D - F - C -$ 34, from correct working seen Correctly explaining why method fails, need to have explicitly considered both cases	[4]
Total = 10					

For reference

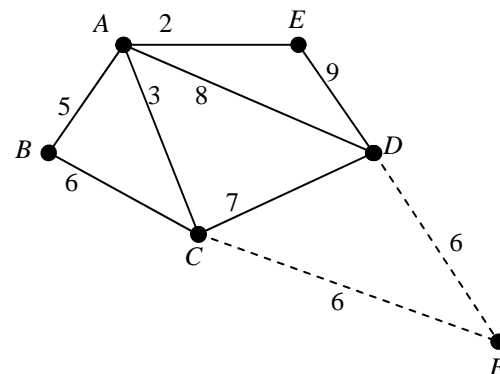
(ii)



(iii) (iv)

	A	B	C	D	E
A	-	5	3	8	2
B	5	-	6	-	-
C	3	6	-	7	-
D	8	-	7	-	9
E	2	-	-	9	-

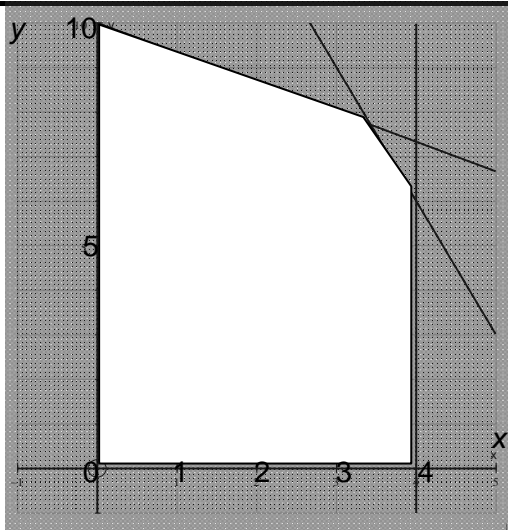
$CF = 6$
 $DF = 6$



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3 (i)	x = number of clients who use program X y = number of clients who use program Y	B1	Number of clients on X and Y , respectively	[1]
(ii)	Spin cycle: $30x + 10y \leq 180$ $\Rightarrow 3x + y \leq 18$ Rower: $10x \leq 40$ $\Rightarrow x \leq 4$ Free weights: $20x + 30y \leq 300$ $\Rightarrow 2x + 3y \leq 30$	B1 B1 B1	$3x + y \leq 18$, or equivalent, simplified $x \leq 4$, or equivalent, simplified $2x + 3y \leq 30$, or equivalent, simplified Allow use of slack variables instead of inequalities	[3]
(iii)	Both must take non-negative integer values	B1	Non-negative <u>and</u> integer Accept $x + y \leq 12$ as an alternative answer	[1]
(iv)	 <p>Checking vertices or using a profit line $(4, 6) \rightarrow 72$ $(3\frac{3}{7}, 7\frac{5}{7}) \rightarrow 77\frac{1}{7}$ or $(24/7, 54/7) \rightarrow 77\frac{1}{7}$ $(0, 10) \rightarrow 60$ $(4, 0) \rightarrow 36$</p> <p>Checking other feasible integer points near (non-integer) optimum for continuous problem $(3, 8) \rightarrow 75$</p> <p>Put 3 clients on program X, 8 on program Y and 1 on program Z</p>	B1 M1 A1 M1 M1 A1	<p>Axes scaled and labelled appropriately (on graph paper)</p> <p>Boundaries of their three constraints shown correctly (non-negativity may be missed)</p> <p>Correct graph with correct shading or feasible region correct and clearly identified (non-negativity may be missed) (cao)</p> <p>Follow through their graph if possible $x = 3.4, y = 7.7$ may be implied from $(3, 8)$</p> <p>Could be implied from identifying point $(3, 8)$ in any form</p> <p>cao, in context and including program Z</p>	[3]
Total =				11

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4	(i)	<table><tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>D</td><td>D</td><td>D</td><td>D</td><td>C</td></tr><tr><td>C</td><td>C</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td></tr></table>	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	D	D	D	D	C	C	C	B	B	B	B	B	B	B	B	B1	15 A's, 4 D's, 3 C's, 8B's (but not just A D C B)																													
		A	A	A	A	A	A	A	A	A	A																																																				
A	A	A	A	A	D	D	D	D	C																																																						
C	C	B	B	B	B	B	B	B	B																																																						
<table><tr><td>Box 1</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>Box 2</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>Box 3</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>Box 4</td><td>D</td><td>D</td><td>D</td><td>D</td><td>C</td><td>C</td><td>C</td><td>B</td></tr><tr><td></td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td></td></tr></table> <p>Cannot fit all the items into box 4 There is only room for one B in a box</p>	Box 1	A	A	A	A	A	Box 2	A	A	A	A	A	Box 3	A	A	A	A	A	Box 4	D	D	D	D	C	C	C	B		B	B	B	B	B	B	B		M1 M1 A1 B1	Three boxes each containing A A A A A (or shown using weights) A box containing all the rest Completely correct, including order of packing into boxes Any identification of a (specific) volume conflict	[5]																								
Box 1	A	A	A	A	A																																																										
Box 2	A	A	A	A	A																																																										
Box 3	A	A	A	A	A																																																										
Box 4	D	D	D	D	C	C	C	B																																																							
	B	B	B	B	B	B	B																																																								
	(ii)	<table><tr><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>B</td><td>C</td><td>C</td></tr><tr><td>C</td><td>D</td><td>D</td><td>D</td><td>D</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr></table>	B	B	B	B	B	B	B	B	C	C	C	D	D	D	D	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	B1	8 B's, 3 C's, 4 D's, 15 A's (but not just B C D A)																													
		B	B	B	B	B	B	B	B	C	C																																																				
C	D	D	D	D	A	A	A	A	A																																																						
A	A	A	A	A	A	A	A	A	A																																																						
<table><tr><td>Box 1</td><td>B</td><td>D</td><td>A</td><td>A</td></tr><tr><td>Box 2</td><td>B</td><td>D</td><td>A</td><td>A</td></tr><tr><td>Box 3</td><td>B</td><td>D</td><td>A</td><td>A</td></tr><tr><td>Box 4</td><td>B</td><td>D</td><td>A</td><td>A</td></tr><tr><td>Box 5</td><td>B</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td><td>A</td></tr><tr><td>Box 6</td><td>B</td><td>A</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>Box 7</td><td>B</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>Box 8</td><td>B</td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>Box 9</td><td>C</td><td>C</td><td>C</td><td></td><td></td><td></td><td></td></tr></table> <p>Box 5 is over the weight limit More than five A's is too heavy for one box</p>	Box 1	B	D	A	A	Box 2	B	D	A	A	Box 3	B	D	A	A	Box 4	B	D	A	A	Box 5	B	A	A	A	A	A	A	Box 6	B	A						Box 7	B							Box 8	B							Box 9	C	C	C					M1 M1 A1 B1	Four boxes each containing B D A A (in any order) Using exactly 9 boxes, the first eight of which each contain a B (with or without other items) and the ninth contains three C's. Completely correct, including order of packing into boxes Any identification of a (specific) weight conflict	[5]
Box 1	B	D	A	A																																																											
Box 2	B	D	A	A																																																											
Box 3	B	D	A	A																																																											
Box 4	B	D	A	A																																																											
Box 5	B	A	A	A	A	A	A																																																								
Box 6	B	A																																																													
Box 7	B																																																														
Box 8	B																																																														
Box 9	C	C	C																																																												
	(iii)	Items may be the wrong shape for the boxes eg too tall	B1	Reference to shape, height, etc. but not practical issues connected with the food	[1]																																																										
Total = 11																																																															

For reference

Item type	A	B	C	D
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm ³)	1 000	24 000	10 000	4 000
Weight (g)	1 000	250	300	400

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5	(i)	<p>Minimise $2a - 3b + c + 18$ \Rightarrow minimise $2(20-x) - 3(10-y) + (8-z) + 18$ \Rightarrow minimise $-2x + 3y - z$ \Rightarrow maximise $2x - 3y + z$ (given)</p> <p>$a + b - c \geq 14$ $\Rightarrow (20-x) + (10-y) - (8-z) \geq 14$ $\Rightarrow x + y - z \leq 8$ (given)</p> <p>$-2a + 3c \leq 50$ $\Rightarrow -2(20-x) + 3(8-z) \leq 50$ $\Rightarrow 2x - 3z \leq 66$ (given)</p> <p>$10 + 4a \geq 5b$ $\Rightarrow 10 + 4(20-x) \geq 5(10-y)$ $\Rightarrow 4x - 5y \leq 40$ (given)</p> <p>$a \leq 20 \Rightarrow 20-x \leq 20 \Rightarrow x \geq 0$ $b \leq 10 \Rightarrow 10-y \leq 10 \Rightarrow y \geq 0$ $c \leq 8 \Rightarrow 8-z \leq 8 \Rightarrow z \geq 0$</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>(Constant has no effect on slope of objective) Replacing a, b and c in objective to get $-2x + 3y - z$ (Condone omission of conversion to maximisation here)</p> <p>Replacing a, b and c in the first three constraints to get given expressions</p> <p>Showing how $a \leq 20, b \leq 10, c \leq 8$ give $x \geq 0, y \geq 0, z \geq 0$</p>	[3]																																																																																
	(ii)	<table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr><tr><td>1</td><td>-2</td><td>3</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>8</td></tr><tr><td>0</td><td>2</td><td>0</td><td>-3</td><td>0</td><td>1</td><td>0</td><td>66</td></tr><tr><td>0</td><td>4</td><td>-5</td><td>0</td><td>0</td><td>0</td><td>1</td><td>40</td></tr></table> <p>x and z columns have negative entries in objective row, but z column has no positive entries in constraint rows, so pivot on x col $8 \div 1 = 8; 66 \div 2 = 33; 40 \div 4 = 10$ Least ratio is $8 \div 1$, so pivot on 1 from x col</p> <p>New row 2 = row 2 New row 1 = row 1 + 2(new row 2) New row 3 = row 3 - 2(new row 2) New row 4 = row 4 - 4(new row 2)</p> <table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr><tr><td>1</td><td>0</td><td>5</td><td>-3</td><td>2</td><td>0</td><td>0</td><td>16</td></tr><tr><td>0</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>8</td></tr><tr><td>0</td><td>0</td><td>-2</td><td>-1</td><td>-2</td><td>1</td><td>0</td><td>50</td></tr><tr><td>0</td><td>0</td><td>-9</td><td>4</td><td>-4</td><td>0</td><td>1</td><td>8</td></tr></table> <p>$x = 8, y = 0, z = 0$</p> <p>$x = 8 \Rightarrow a = 20 - 8 = 12$ $y = 0 \Rightarrow b = 10 - 0 = 10$ $z = 0 \Rightarrow c = 8 - 0 = 8$</p>	P	x	y	z	s	t	u	RHS	1	-2	3	-1	0	0	0	0	0	1	1	-1	1	0	0	8	0	2	0	-3	0	1	0	66	0	4	-5	0	0	0	1	40	P	x	y	z	s	t	u	RHS	1	0	5	-3	2	0	0	16	0	1	1	-1	1	0	0	8	0	0	-2	-1	-2	1	0	50	0	0	-9	4	-4	0	1	8	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>Constraint rows correct, with three slack variable columns Objective row correct</p> <p>Choosing to pivot on x column (may be implied from pivot choice)</p> <p>Calculations seen or referred to and correct pivot choice made (cao)</p> <p>Pivot row unchanged (may be implied) or follow through for their +ve pivot</p> <p>Calculations for other rows shown (cao)</p> <p>An augmented tableau with three basis columns, non-negative values in final column and value of objective having not decreased</p> <p>Correct tableau after one iteration (cao)</p> <p>Non-negative values for x, y and z from their tableau</p> <p>Putting their values for x, y and z into $a = 20 - x, b = 10 - y$ and $c = 8 - z$</p> <p>Correct values for a, b and c, from their non-negative x, y and z</p>	<p>[2]</p> <p>[2]</p> <p>[2]</p> <p>[2]</p> <p>[3]</p>
P	x	y	z	s	t	u	RHS																																																																														
1	-2	3	-1	0	0	0	0																																																																														
0	1	1	-1	1	0	0	8																																																																														
0	2	0	-3	0	1	0	66																																																																														
0	4	-5	0	0	0	1	40																																																																														
P	x	y	z	s	t	u	RHS																																																																														
1	0	5	-3	2	0	0	16																																																																														
0	1	1	-1	1	0	0	8																																																																														
0	0	-2	-1	-2	1	0	50																																																																														
0	0	-9	4	-4	0	1	8																																																																														
	(iii)	$x \leq 20, y \leq 10$ and $z \leq 8$	<p>M1</p> <p>A1</p>	<p>20, 10, 8</p> <p>Correct inequalities for x, y and z</p>	[2]																																																																																
Total = 16																																																																																					

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TO BE ANSWERED ON INSERT																																												
6	(i)	10 $\frac{1}{2}n(n-1)$	B1 B1	10 $1+2+\dots+(n-1)$ seen, or equivalent Check that sum stops at $n-1$ not n	[2]																																							
	(ii)(a)	9 1 2 3 45	B1 M1 A1	Their 10 minus 1 1, 2 and 3 45 following from method mark earned cao	[3]																																							
	(b)	$1+2+3+\dots+(N-1)$ $= \frac{1}{2}N(N-1)$, where $N = \frac{1}{2}n(n-1)$ $= \frac{1}{4}n(n-1)(\frac{1}{2}n(n-1) - 1)$ (given)	M1 A1	$1+2+3+\dots+(N-1)$ or $\frac{1}{2}N(N-1)$, where $N = \frac{1}{2}n(n-1)$ Convincingly achieving the given result	[2]																																							
	(iii)	<table><tr><th>M1 Vertices in tree</th><th>M2 Arcs in tree</th><th>M3 Vertices not in tree</th></tr><tr><td></td><td></td><td>ABCDE</td></tr><tr><td>D E</td><td>D 2 E</td><td>A B C</td></tr><tr><td>D E A</td><td>D 2 E A 3 E</td><td>B C</td></tr><tr><td>D E A C</td><td>D 2 E A 3 E A 4 C</td><td>B</td></tr><tr><td>DEACB</td><td>D 2 E A 3 E A 4 C B 6 E</td><td></td></tr></table>	M1 Vertices in tree	M2 Arcs in tree	M3 Vertices not in tree			ABCDE	D E	D 2 E	A B C	D E A	D 2 E A 3 E	B C	D E A C	D 2 E A 3 E A 4 C	B	DEACB	D 2 E A 3 E A 4 C B 6 E		<table><tr><th>M4 Sorted list</th></tr><tr><td>D 2 E</td></tr><tr><td>A 3 E</td></tr><tr><td>A 4 C</td></tr><tr><td>C 5 D</td></tr><tr><td>B 6 E</td></tr><tr><td>B 7 C</td></tr><tr><td>A 8 B</td></tr><tr><td>C 9 E</td></tr></table>	M4 Sorted list	D 2 E	A 3 E	A 4 C	C 5 D	B 6 E	B 7 C	A 8 B	C 9 E	M1 Arc <table><tr><td>A</td><td>3</td><td>E</td></tr></table> is added to M2, A is added to M1 and deleted from M3 M1 Arc <table><tr><td>A</td><td>4</td><td>C</td></tr></table> is added to M2, C is added to M1 and deleted from M3 M1 Arc <table><tr><td>C</td><td>5</td><td>D</td></tr></table> is not added to M2 and arc <table><tr><td>B</td><td>6</td><td>E</td></tr></table> is added to M2 A1 cao (lists M1, M2 and M3 totally correct, ignore what is done in list M4).	A	3	E	A	4	C	C	5	D	B	6	E	[4]
M1 Vertices in tree	M2 Arcs in tree	M3 Vertices not in tree																																										
		ABCDE																																										
D E	D 2 E	A B C																																										
D E A	D 2 E A 3 E	B C																																										
D E A C	D 2 E A 3 E A 4 C	B																																										
DEACB	D 2 E A 3 E A 4 C B 6 E																																											
M4 Sorted list																																												
D 2 E																																												
A 3 E																																												
A 4 C																																												
C 5 D																																												
B 6 E																																												
B 7 C																																												
A 8 B																																												
C 9 E																																												
A	3	E																																										
A	4	C																																										
C	5	D																																										
B	6	E																																										
	(iv)	$30 \times \left(\frac{500}{100}\right)^4$ $= 18750$ seconds	M1 A1	Or equivalent cao, with units (312 min 30 sec or 5 hours 12 min 30 sec)	[2]																																							
Total =					13																																							