

Mark Scheme 4733 January 2006

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1	(i) (a) Po(2): $1 - P(\leq 3) = 0.1429$	M1 A1	2	Po(2) tables, "1 -" used Answer, a.r.t. 0.143
	(b) Po(2/3): $e^{-2/3} \frac{(2/3)^2}{2!} = 0.114$	M1 M1 A1	3	Parameter 2/3 Poisson formula correct, $r = 2$, any μ Answer, a.r.t. 0.114
	(ii) Foxes may congregate so not independent	B1 B1	2	Independent/not constant rate/singly used Any valid relevant application in context
2	N(80/7, 400/49) $\frac{13.5 - \frac{80}{7}}{\frac{20}{7}} = 0.725$ $1 - \Phi(0.725) = 0.2343$	B1 B1 M1 A1 A1 M1 A1	7	80/7, a.e.f (11.43) 400/49 or 20/7 seen, a.e.f. (8.163 or 2.857) Standardise with np & npq or \sqrt{npq} or nq , no \sqrt{n} \sqrt{npq} correct 13.5 correct Normal tables used, answer < 0.5 Answer, a.r.t. 0.234 [SR: Binomial, complete expression M1, 0.231 A1 Po(80/7) B1, complete expression M1, 0.260 A1 Normal approx to Poisson, B1B0 M1A0A1 M1A0]
3	$H_0: p = 0.3$ $H_1: p \neq 0.3$ B(8, 0.3) $P(\leq 4) = 0.9420$; $P(> 4) = 0.0580$ $P(\leq 5) = 0.9887$; $P(> 5) = 0.0113$ Compare 0.025 or critical value 6 Do not reject H_0 Insufficient evidence that manufacturer's claim is wrong	B1 B1 M1 A1 M1 M1 A1 $\sqrt{}$	7	NH stated, must be this form (or π) AH stated, must be this form (or π) [μ : B1 both] B(8, 0.3) stated or implied Any one of these four probabilities seen <i>Either</i> compare $P(\geq 5)$ & 0.025 / $P(\leq 4)$ & 0.975 <i>Or</i> critical region ≥ 6 with 5 H_0 not rejected, can be implied, needs essentially correct method Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 $P(\leq 5)$: first 4 marks. $P(= 5)$: first 3 marks only.]
4	(i) B(80, 0.02) approx Po(1.6) $1 - P(\leq 1) = 1 - 0.5249 = 0.4751$	M1 M1 M1 A1	4	B(80, 0.02) seen or implied, e.g. N(1.6, 1.568) Po(np) used $1 - P(\leq 1)$ used Answer, a.r.t. 0.475 [SR: Exact: M1 M0 M0, 0.477 A1]
	(ii) $P(\leq 4) = 0.9763$, $P(\geq 5) = 0.0237$ $P(\leq 5) = 0.9940$, $P(\geq 6) = 0.0060$ Therefore least value is 6	M1 A1 A1	3	Evidence for correct method, e.g. answer 6 At least one of these probabilities seen Answer 6 only [SR N(1.6,1.568): $2.326 = (r - 1.6)/\sqrt{1.568}$ M1 $r = 5$ or (with cc) 6 A1 Exact: M1 A0 A1]

5	(i)	$\frac{0 - \mu}{\mu/2} = -2$, independent of μ $1 - \Phi(2) = 1 - 0.9772 = 0.0228$	M1 A1 A1 A1	4	Standardise, allow $-$, allow $\mu^2/4$ $z = 2$ or -2 z -value independent of μ and any relevant statement Answer, a.r.t. 0.023
	(ii)	$\Phi[(9 - 6)/3]$ $\Phi(1.0) = 0.8413$ $[\Phi(1.0)]^3 = 0.59546$	M1 A1 M1 A1	4	Standardise and use Φ [no \sqrt{n}] 0.8413 [not 0.1587] Cube previous answer Answer, in range [0.595, 0.596]
	(iii)	Annual increases not independent	B1	1	Independence mentioned, in context. Allow "one year affects the next" but not "years not random"
6		$H_0: \mu = 32; H_1: \mu > 32$, where μ is population mean waist measurement $\bar{W} = 32.3$ $s^2 = 52214.50/50 - \bar{W}^2$ [= 1] $\hat{\sigma}^2 = 50/49 \times s^2$ [= 50/49 or 1.0204]	B1 B1 B1 M1 M1		One hypothesis correctly stated, <i>not</i> x or \bar{x} or \bar{w} Both completely correct, μ used Sample mean 32.3 seen Correct formula for s^2 used Multiply by 50/49 or $\sqrt{\quad}$
	$\alpha:$	$z = (32.3 - 32) \times \sqrt{49} = 2.1$ Compare 2.1 with 3.09 or 0.0179 with 0.001	M1 A1 B1		Correct formula for z , can use s , aef, need $\mu = 32$ $z = 2.1$ or $1 - \Phi(z) = 0.0179$, <i>not</i> -2.1 Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001
	$\beta:$	$CV = 32 + 3.09 \div \sqrt{49} = 32.44$ Compare CV with 32.3	M1 B1 A1 $\sqrt{\quad}$		$32 + z \times \sigma/\sqrt{n}$ [allow \pm , s , any z] $z = 3.09$ and (later) compare \bar{x} CV in range [32.4, 32.5], $\sqrt{\quad}$ on k
		Do not reject H_0 Insufficient evidence that waists are actually larger	M1 $\sqrt{\quad}$ A1 $\sqrt{\quad}$	10	Correct conclusion, can be implied, needs essentially correct method including \sqrt{n} , any reasonable σ , but not from $\mu = 32.3$ Interpreted in context
7	(i)	$\frac{80 - c}{8/\sqrt{12}} = 2.326$ $c = 74.63$	M 1 A 1 B 1 A 1	4	Equate standardised variable to Φ^{-1} , allow $-$ $\sqrt{12}$, 8 correct 2.326 or a.r.t 2.33 seen, signs must be correct Answer, a.r.t. 74.6, cwo, allow \leq or \geq
	(ii)	(a) Type I error (b) Correct	B 1 $\sqrt{\quad}$ B 1 $\sqrt{\quad}$	1 1	"Type I error" stated, needs evidence "Correct" stated or clearly implied Wrong c : $74 < c < 75$, B1 $\sqrt{\quad}$ B1 $\sqrt{\quad}$ $c < 74$, both "correct", B1. $75 < c < 80$, both "Type I", B1 Also allow if only one is answered
	(iii)	$\frac{74.63 - \mu}{8/\sqrt{12}} = -1.555$ Solve for μ $\mu = 78.22$	M1*d ep A1 $\sqrt{\quad}$ dep* M1 A1	4	$\frac{c - \mu}{8/\sqrt{12}} = (\pm)\Phi^{-1}$, allow no $\sqrt{12}$ but not 80, not 0.8264 Correct including sign, $\sqrt{\quad}$ on their c Solve to find μ , dep, answer consistent with signs Answer, a.r.t. 78.2

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8	(i) $\int_0^1 x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_0^1 = \frac{1}{n+1}$ $k/(n+1) = 1$ so $k = n+1$	M1 M1 A1 3	Integrate x^n , limits 0 and 1 Equate to 1 and solve for k Answer $n+1$, <i>not</i> 1^{n+1} , c.w.o.
	(ii) $\int_0^1 x^{n+1} dx = \left[\frac{x^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+2}$ $\mu = \frac{k}{n+2} = \frac{n+1}{n+2}$ AG	M1 A1 A1 3	Integrate x^{n+1} , limits 0 and 1, not just $x \cdot x^n$ Answer $\frac{1}{n+2}$ Correctly obtain given answer
	(iii) $\int_0^1 x^5 dx = \left[\frac{x^6}{6} \right]_0^1 [= \frac{1}{6}]$ $\sigma^2 = \frac{4}{6} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$	M1 M1 A1 3	Integrate x^5 , limits 0 and 1, allow with n Subtract $\left(\frac{4}{5}\right)^2$ Answer $\frac{2}{75}$ or a.r.t. 0.027
	(iv) $N\left(\frac{4}{5}, \frac{2}{7500}\right)$	B1 B1 B1√ 3	Normal stated Mean $\frac{4}{5}$ or $\frac{n+1}{n+2}$ Variance their (iii)/100, a.e.f., allow √
	(v) Same distribution, translated Mean 0 Variance $\frac{2}{75}$	M1 A1√ B1√ 3	Can be negative translation; <i>or</i> integration, must include correct method for integral (Their mean) $- \frac{4}{5}$, c.w.d. Variance same as their (iii), or $\frac{2}{75}$ by integration