

4722 Core Mathematics 2

<p>1 (i) $\cos \theta = \frac{6.4^2 + 7.0^2 - 11.3^2}{2 \times 6.4 \times 7.0}$ $= -0.4211$ $\theta = 115^\circ$ or 2.01 rads</p>	<p>M1 Attempt use of cosine rule (any angle) A1 Obtain one of 115°, 34.2°, 30.9°, 2.01, 0.597, 0.539 A1 3 Obtain 115° or 2.01 rads, or better</p>
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<p>(ii) area = $\frac{1}{2} \times 7 \times 6.4 \times \sin 115$ $= 20.3 \text{ cm}^2$</p>	<p>M1 Attempt triangle area using $(\frac{1}{2})ab \sin C$, or equiv A1 2 Obtain 20.3 (cao)</p>
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<p>2 (i) $a + 9d = 2(a + 3d)$ $a = 3d$ $a + 19d = 44 \Rightarrow 22d = 44$ $d = 2, a = 6$</p>	<p>M1* Attempt use of $a + (n - 1)d$ or $a + nd$ at least once for u_4, u_{10} or u_{20} A1 Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$ M1dep* Attempt to eliminate one variable from two simultaneous equations in a and d, from u_4, u_{10}, u_{20} and no others A1 4 Obtain $d = 2, a = 6$</p>
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<p>(ii) $S_{50} = \frac{50}{2} (2 \times 6 + 49 \times 2)$ $= 2750$</p>	<p>M1 Attempt S_{50} of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$ A1 2 Obtain 2750</p>
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<p>3 $\log 7^x = \log 2^{x+1}$ $x \log 7 = (x + 1) \log 2$ $x(\log 7 - \log 2) = \log 2$ $x = 0.553$</p>	<p>M1 Introduce logarithms throughout, or equiv with base 7 or 2 M1 Drop power on at least one side A1 Obtain correct linear equation (allow with no brackets) M1 Either expand bracket and attempt to gather x terms, or deal correctly with algebraic fraction A1 5 Obtain $x = 0.55$, or rounding to this, with no errors seen</p>
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<p>4 (i) $(x^2 - 5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$ $= x^6 - 15x^4 + 75x^2 - 125$</p> <p>OR</p> <p>$(x^2 - 5)^3 = (x^2 - 5)(x^4 - 10x^2 + 25)$ $= x^6 - 15x^4 + 75x^2 - 125$</p>	<p>M1* Attempt expansion, with product of powers of x^2 and ± 5, at least 3 terms M1* Use at least 3 of binomial coeffs of 1, 3, 3, 1 A1dep* Obtain at least two correct terms, coeffs simplified A1 4 Obtain fully correct expansion, coeffs simplified</p> <p>M2 Attempt full expansion of all 3 brackets A1 Obtain at least two correct terms A1 Obtain full correct expansion</p>
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<p>(ii) $\int (x^2 - 5)^3 dx = \frac{1}{7} x^7 - 3x^5 + 25x^3 - 125x + c$</p>	<p>M1 Attempt integration of terms of form kx^n A1√ Obtain at least two correct terms, allow unsimplified coeffs A1 Obtain $\frac{1}{7} x^7 - 3x^5 + 25x^3 - 125x$ B1 4 $+ c$, and no dx or \int sign</p>
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<p>5 (i) $2x = 30^\circ, 150^\circ$ $x = 15^\circ, 75^\circ$</p>	<p>M1 Attempt $\sin^{-1} 0.5$, then divide or multiply by 2 A1 Obtain 15° (allow $\pi/12$ or 0.262) A1 3 Obtain 75° (not radians), and no extra solutions in range</p>

<p>(ii) $2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$ $2\cos^2 x - \sqrt{3}\cos x = 0$ $\cos x (2\cos x - \sqrt{3}) = 0$ $\cos x = 0, \cos x = \frac{1}{2}\sqrt{3}$ range $x = 90^\circ, x = 30^\circ$</p>	<p>M1 Use $\sin^2 x = 1 - \cos^2 x$ A1 Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms) M1 Attempt to solve quadratic in $\cos x$ A1 Obtain 30° (allow $\pi/6$ or 0.524), and no extra solns in B1 5 Obtain 90° (allow $\pi/2$ or 1.57), from correct quadratic only SR answer only B1 one correct solution B1 second correct solution, and no others</p>

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<p>6 $\int (3x^2 + a) dx = x^3 + ax + c$ $(-1, 2) \Rightarrow -1 - a + c = 2$ $(2, 17) \Rightarrow 8 + 2a + c = 17$ $a = 2, c = 5$ Hence $y = x^3 + 2x + 5$</p>	<p>M1 Attempt to integrate A1 Obtain at least one correct term, allow unsimplified A1 Obtain $x^3 + ax$ M1 Substitute at least one of $(-1, 2)$ or $(2, 17)$ into integration attempt involving a and c A1 Obtain two correct equations, allow unsimplified M1 Attempt to eliminate one variable from two equations in a and c A1 Obtain $a = 2, c = 5$, from correct equations A1 8 State $y = x^3 + 2x + 5$</p>
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<p>7 (i) $f(-2) = -16 + 36 - 22 - 8$ $= -10$</p>	<p>M1 Attempt $f(-2)$, or equiv A1 2 Obtain -10</p>

<p>(ii) $f(\frac{1}{2}) = \frac{1}{4} + 2\frac{1}{4} + 5\frac{1}{2} - 8 = 0$ AG</p>	<p>M1 Attempt $f(\frac{1}{2})$ (no other method allowed) A1 2 Confirm $f(\frac{1}{2}) = 0$, extra line of working required</p>

<p>(iii) $f(x) = (2x - 1)(x^2 + 5x + 8)$</p>	<p>M1 Attempt complete division by $(2x - 1)$ or $(x - \frac{1}{2})$ or equiv A1 Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$ A1 3 State $(2x - 1)(x^2 + 5x + 8)$ or $(x - \frac{1}{2})(2x^2 + 10x + 16)$</p>

<p>(iv) $f(x)$ has one real root ($x = \frac{1}{2}$) because $b^2 - 4ac = 25 - 32 = -7$ hence quadratic has no real roots as $-7 < 0$,</p>	<p>B1√ State 1 root, following their quotient, ignore reason B1√ 2 Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at $(-2.15, -9.9)$</p>

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<p>8 (i) $\frac{1}{2} \times r^2 \times 1.2 = 60$ $r = 10$ $r\theta = 10 \times 1.2 = 12$ perimeter = $10 + 10 + 12 = 32$ cm</p>	<p>M1 Attempt $(\frac{1}{2}) r^2\theta = 60$ A1 Obtain $r = 10$ B1√ State or imply arc length is $1.2r$, following their r A1 4 Obtain 32</p>
<p>(ii)(a) $u_5 = 60 \times 0.6^4$ $= 7.78$</p>	<p>M1 Attempt u_5 using ar^4, or list terms A1 2 Obtain 7.78, or better</p>
<p>(b) $S_{10} = \frac{60(1-0.6^{10})}{1-0.6}$ $= 149$</p>	<p>M1 Attempt use of correct sum formula for a GP, or sum terms A1 2 Obtain 149, or better (allow 149.0 – 149.2 inclusive)</p>
<p>(c) common ratio is less than 1, so series is convergent and hence sum to infinity exists</p> <p>$S_\infty = \frac{60}{1-0.6}$ $= 150$</p>	<p>B1 series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time</p> <p>M1 Attempt S_∞ using $\frac{a}{1-r}$ A1 3 Obtain $S_\infty = 150$</p> <p>SR B1 only for 150 with no method shown</p>

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<p>9 (i)</p> 	<p>B1 Sketch graph showing exponential growth (both quadrants) B1 2 State or imply (0, 4)</p>
<p>(ii) $4k^x = 20k^2$ $k^x = 5k^2$ $x = \log_k 5k^2$ $x = \log_k 5 + \log_k k^2$ $x = 2\log_k k + \log_k 5$ $x = 2 + \log_k 5$ AG</p> <p>OR $4k^x = 20k^2$ $k^x = 5k^2$ $k^{x-2} = 5$ $x - 2 = \log_k 5$ $x = 2 + \log_k 5$ AG</p>	<p>M1 Equate $4k^x$ to $20k^2$ and take logs (any, or no, base) M1 Use $\log ab = \log a + \log b$ M1 Use $\log a^b = b \log a$ A1 4 Show given answer correctly</p> <p>M1 Attempt to rewrite as single index A1 Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$ M1 Take logs (to any base) A1 Show given answer correctly</p>
<p>(iii) (a) area $\approx \frac{1}{2} \times \frac{1}{2} \times (4k^0 + 8k^{\frac{1}{2}} + 4k^1)$ $\approx 1 + 2k^{\frac{1}{2}} + k$</p>	<p>M1 Attempt y-values at $x = 0, \frac{1}{2}$ and 1, and no others M1 Attempt to use correct trapezium rule, 3 y-values, $h = \frac{1}{2}$ A1 3 Obtain a correct expression, allow unsimplified</p>
<p>(b) $1 + 2k^{\frac{1}{2}} + k = 16$ $(k^{\frac{1}{2}} + 1)^2 = 16$ $k^{\frac{1}{2}} = 3$ $k = 9$</p>	<p>M1 Equate attempt at area to 16 M1 Attempt to solve ‘disguised’ 3 term quadratic A1 3 Obtain $k = 9$ only</p>

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