Mark Scheme (Final)
Summer 2007

GCE

GCE Mathematics (6681/01)

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6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathbf{d} = \mathbf{AB} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ $\frac{1}{2}.0.5v^2 = \mathbf{F}.\mathbf{d} = (4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}).(3\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ $v = 6 \text{ m s}^{-1}$	B1 M1 A1 A1 **(4)**
2.	$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + 3\mathbf{v} = \mathbf{0}$ $\text{IF} = \mathrm{e}^{3t} \Rightarrow \frac{\mathrm{d}(\mathbf{v}\mathrm{e}^{3t})}{\mathrm{d}t} = \mathbf{0}$ $\Rightarrow \mathbf{v}\mathrm{e}^{3t} = \mathbf{A}$ $t = 0, \mathbf{v} = 8\mathbf{i} - 12\mathbf{j} \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})\mathrm{e}^{-3t}$ $t = \frac{2}{3}\ln 2 \Rightarrow \mathbf{v} = (8\mathbf{i} - 12\mathbf{j})\mathrm{e}^{-2\ln 2} = (2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$	B1 M1 A1 M1 A1 DM1 A1 **(7)**
3.	$\frac{4}{3}ma^2\omega = (\frac{4}{3}ma^2 + mx^2)\frac{3}{4}\omega$ $\Rightarrow x = \frac{2}{3}a$	M1A1A1 DM1A1 **(5)**
4.	$V = \pi\int_0^a 4ax \ \mathrm{d}x$ $= 2\pi a^3$ $\delta m = \frac{M}{2\pi a^3}.\pi 4ax\delta x \quad (= \frac{2M}{a^2}x\delta x)$ $\delta I = \frac{1}{2}\frac{2M}{a^2}x\delta x.y^2 = \frac{4M}{a}x^2\delta x$ $I = \frac{4M}{a}\int_0^a x^2 \ \mathrm{d}x = \frac{4}{3}Ma^2$	M1 A1 M1 M1A1 DM1A1 **(7)**

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5. (a)	$\mathbf{F} = \Sigma\, \mathbf{F}_i = (8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ N	B1 (1)
(b)	$\Sigma\, \mathbf{r}_i \times \mathbf{F}_i = (\mathbf{i} - 2\mathbf{j})\text{x}(3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) + (3\mathbf{i} - \mathbf{k})\text{x}(5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1
	$= (12\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}) + (-\mathbf{i} - 11\mathbf{j} - 3\mathbf{k}) \quad (= (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	A2 , 1 , 0
	$\mathbf{G} + (\mathbf{i} - \mathbf{k})\text{x}(8\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = (11\mathbf{i} - 5\mathbf{j} + 7\mathbf{k})$	M1
	$(3\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}),\ \mathbf{G} = (8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$	B1, A1
	$\mathbf{G} = \sqrt{8^2 + (-1)^2 + 4^2} = 9$ Nm.	M1A1 (8)
		(9)
6. (a)	$I_O = \dfrac{M}{3\pi a^2}\left(\tfrac{\pi}{2}(2a)^2(2a)^2 - \tfrac{\pi}{2}(a)^2(a)^2\right)$	M1 A1
	$= \dfrac{5Ma^2}{2}$ *	A1 (3)
(b)	$I_{diameter} = \tfrac{1}{2}\dfrac{5Ma^2}{2}$ (perp. axes)	M1 A1
	$I_L = \dfrac{5Ma^2}{4} + M(2a)^2$ (parallel axes)	M1
	$= \dfrac{21Ma^2}{4}$	A1 (4)
(c)	M(L), $-Mg2a\sin\theta = \dfrac{21Ma^2}{4}\ddot{\theta}$	M1 A1
	$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -\dfrac{8g}{21a}\theta$, so SHM	DM1 A1
	Time$= \tfrac{1}{4}2\pi\sqrt{\dfrac{21a}{8g}}$	DM1
	$= \tfrac{\pi}{2}\sqrt{\dfrac{21a}{8g}}$	A1 (6)
		(13)

7.(a)	$(m+\delta m)(v+\delta v)-mv=-2\lambda v\,\delta t$	M1 A1
	$m\dfrac{\mathrm{d}v}{\mathrm{d}t}+v\dfrac{\mathrm{d}m}{\mathrm{d}t}=-2\lambda v$	
	$\dfrac{\mathrm{d}m}{\mathrm{d}t}=\lambda;\quad m=M+\lambda t$	B1 ; B1
	$(M+\lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t}+3\lambda v=0\quad *$	D M1 A1
		(6)
(b)	$-\displaystyle\int\frac{\mathrm{d}v}{3\lambda v}=\int\frac{\mathrm{d}t}{(M+\lambda t)}$	M1
	$-\dfrac{1}{3\lambda}\left[\ln v\right]_{u}^{\frac{1}{2}u}=\dfrac{1}{\lambda}\left[\ln(M+\lambda t)\right]_{0}^{T}$	DM1 A1
	$\dfrac{1}{3}\ln 2=\ln\dfrac{(M+\lambda T)}{M}$	DM1
	$T=\dfrac{M}{\lambda}(2^{\frac{1}{3}}-1)*$	DM1 A1
		(6)
(c)	Sinks at $T_S=\dfrac{M}{\lambda}$	
	Reaches speed $\dfrac{1}{2}U$ at $T=\dfrac{M}{\lambda}(2^{\frac{1}{3}}-1)$	
	Since $(2^{\frac{1}{3}}-1)<1,\quad T<T_S$	M1
	i.e. Reaches speed $\dfrac{1}{2}U$ before it sinks	A1c.s.o.
		(2)
		(14)

8.(a)	MI of rod + particle $=\frac{1}{12}3m(2a)^2+3m(\frac{1}{2}a)^2,+m(\frac{3}{2}a)^2$ $=4ma^2$ $\frac{1}{2}4ma^2(\omega^2-\frac{g}{a})=3mg\frac{a}{2}(1-\cos\theta)+mg\frac{3a}{2}(1-\cos\theta)$ $\omega^2=\frac{g}{2a}(5-3\cos\theta)$	M1, M1 A1 M1 A **ft** A1 DM1 A1 (8)
(b)	$4ma^2\ddot{\theta}=3mg\frac{1}{2}a\sin\theta+mg\frac{3}{2}a\sin\theta$ $\Rightarrow\ \ddot{\theta}=\frac{3g\sin\theta}{4a}$	M1 A1 **ft** A1 (3)
(c)	$F+4mg\cos\theta=3m\frac{1}{2}a\omega^2+m\frac{3}{2}a\omega^2$ $=3ma\frac{g}{2a}(5-3\cos\theta)$ $\Rightarrow\ F=\frac{mg}{2}(15-17\cos\theta)$ When $\theta=\phi$, $F=0$ $\Rightarrow\cos\phi=\frac{15}{17}$	M1 A1 DM1 DM1 A1 (5) **16**