## 6689

## Edexcel GCE

Decision Mathematics D1
(New Syllabus)
Advanced/Advanced Subsidiary
Thursday 23 May 2002 - Afternoon Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes

Materials required for examination Nil

Items included with question papers
Answer booklet

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

## Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.
1.

| Ashford | 6 |
| :--- | :---: |
| Colnbrook | 1 |
| Datchet | 18 |
| Feltham | 12 |
| Halliford | 9 |
| Laleham | 0 |
| Poyle | 5 |
| Staines | 13 |
| Wraysbury | 14 |

The table above shows the points obtained by each of the teams in a football league after they had each played 6 games. The teams are listed in alphabetical order. Carry out a quick sort to produce a list of teams in descending order of points obtained.
(5)
2. While solving a maximizing linear programming problem, the following tableau was obtained.

| Basic <br> variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | 0 | 0 | $1 \frac{2}{3}$ | 1 | 0 | $-\frac{1}{6}$ | $\frac{2}{3}$ |
| $y$ | 0 | 1 | $3 \frac{1}{3}$ | 0 | 1 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $x$ | 1 | 0 | -3 | 0 | -1 | $\frac{1}{2}$ | 1 |
| $P$ | 0 | 0 | 1 | 0 | 1 | 1 | 11 |

(a) Explain why this is an optimal tableau.
(b) Write down the optimal solution of this problem, stating the value of every variable.
(c) Write down the profit equation from the tableau. Use it to explain why changing the value of any of the non-basic variables will decrease the value of $P$.
3.

Figure 1


Figure 2


Five members of staff 1, 2, 3, 4 and 5 are to be matched to five jobs $A, B, C, D$ and $E$. A bipartite graph showing the possible matchings is given in Fig. 1 and an initial matching $M$ is given in Fig. 2.

There are several distinct alternating paths that can be generated from $M$. Two such paths are

$$
2-B=4-E
$$

and

$$
2-A=3-D=5-E
$$

(a) Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

Using the maximum matching algorithm and the initial matching $M$,
(b) find two further distinct alternating paths, making your reasoning clear.
4.

## Figure 3



Figure 3 shows the network of paths in a country park. The number on each path gives its length in km . The vertices $A$ and $I$ represent the two gates in the park and the vertices $B, C, D, E, F, G$ and $H$ represent places of interest.
(a) Use Dijkstra’s algorithm to find the shortest route from A to $I$. Show all necessary working in the boxes in the answer booklet and state your shortest route and its length.

The park warden wishes to check each of the paths to check for frost damage. She has to cycle along each path at least once, starting and finishing at $A$.
(b) (i) Use an appropriate algorithm to find which paths will be covered twice and state these paths.
(ii) Find a route of minimum length.
(iii) Find the total length of this shortest route.
5. An algorithm is described by the flow chart below.

(a) Given that $a=645$ and $b=255$, complete the table in the answer booklet to show the results obtained at each step when the algorithm is applied.
(b) Explain how your solution to part (a) would be different if you had been given that $a=255$ and $b=645$.
(c) State what the algorithm achieves.
6.

Figure 4


A building project is modelled by the activity network shown in Fig. 4. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity. The left box entry at each vertex is the earliest event time and the right box entry is the latest event time.
(a) Determine the critical activities and state the length of the critical path.
(b) State the total float for each non-critical activity.
(c) On the grid in the answer booklet, draw a cascade (Gantt) chart for the project.

Given that each activity requires one worker,
(d) draw up a schedule to determine the minimum number of workers required to complete the project in the critical time. State the minimum number of workers.
7. A company wishes to transport its products from 3 factories $F_{1}, F_{2}$ and $F_{3}$ to a single retail outlet $R$. The capacities of the possible routes, in van loads per day, are shown in Fig. 5.

Figure 5

(a) On Diagram 1 in the answer booklet add a supersource $S$ to obtain a capacitated network with a single source and a single sink. State the minimum capacity of each arc you have added.
(b) (i) State the maximum flow along $S F_{1} A B R$ and $S F_{3} C R$.
(ii) Show these maximum flows on Diagram 2 in the answer booklet, using numbers in circles.

Taking your answer to part (b)(ii) as the initial flow pattern,
(c) (i) use the labelling procedure to find a maximum flow from $S$ to $R$.

Your working should be shown on Diagram 3. List each flow-augmenting route you find together with its flow.
(ii) Prove that your final flow is maximal.
8. A chemical company produces two products $X$ and $Y$. Based on potential demand, the total production each week must be at least 380 gallons. A major customer’s weekly order for 125 gallons of $Y$ must be satisfied.

Product $X$ requires 2 hours of processing time for each gallon and product $Y$ requires 4 hours of processing time for each gallon. There are 1200 hours of processing time available each week. Let $x$ be the number of gallons of $X$ produced and $y$ be the number of gallons of $Y$ produced each week.
(a) Write down the inequalities that $x$ and $y$ must satisfy.

It costs $£ 3$ to produce 1 gallon of $X$ and $£ 2$ to produce 1 gallon of $Y$. Given that the total cost of production is $£ C$,
(b) express $C$ in terms of $x$ and $y$.

The company wishes to minimise the total cost.
(c) Using the graphical method, solve the resulting Linear Programming problem. Find the optimal values of $x$ and $y$ and the resulting total cost.
(d) Find the maximum cost of production for all possible choices of $x$ and $y$ which satisfy the inequalities you wrote down in part (a).

## END

