RECOGNISING ACHIEVEMENT

## ADVANCED GCE

## MATHEMATICS

Core Mathematics 4
WEDNESDAY 21 MAY 2008

Afternoon
Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages) List of Formulae (MF1)

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72 .
- You are reminded of the need for clear presentation in your answers.

1 (a) Simplify $\frac{\left(2 x^{2}-7 x-4\right)(x+1)}{\left(3 x^{2}+x-2\right)(x-4)}$.
(b) Find the quotient and remainder when $x^{3}+2 x^{2}-6 x-5$ is divided by $x^{2}+4 x+1$.

2 Find the exact value of $\int_{1}^{\mathrm{e}} x^{4} \ln x \mathrm{~d} x$.

3 The equation of a curve is $x^{2} y-x y^{2}=2$.
(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{y^{2}-2 x y}{x^{2}-2 x y}$.
(ii) (a) Show that, if $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$, then $y=2 x$.
(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the $x$-axis.

4 Relative to an origin $O$, the points $A$ and $B$ have position vectors $3 \mathbf{i}+2 \mathbf{j}+3 \mathbf{k}$ and $\mathbf{i}+3 \mathbf{j}+4 \mathbf{k}$ respectively.
(i) Find a vector equation of the line passing through $A$ and $B$.
(ii) Find the position vector of the point $P$ on $A B$ such that $O P$ is perpendicular to $A B$.

5 (i) Show that $\sqrt{\frac{1-x}{1+x}} \approx 1-x+\frac{1}{2} x^{2}$, for $|x|<1$.
(ii) By taking $x=\frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$.

6 Two lines have equations

$$
\mathbf{r}=\left(\begin{array}{r}
1 \\
0 \\
-5
\end{array}\right)+t\left(\begin{array}{l}
2 \\
3 \\
4
\end{array}\right) \quad \text { and } \quad \mathbf{r}=\left(\begin{array}{r}
12 \\
0 \\
5
\end{array}\right)+s\left(\begin{array}{r}
1 \\
-4 \\
-2
\end{array}\right)
$$

(i) Show that the lines intersect.
(ii) Find the angle between the lines.

7 (i) Show that, if $y=\operatorname{cosec} x$, then $\frac{\mathrm{d} y}{\mathrm{~d} x}$ can be expressed as $-\operatorname{cosec} x \cot x$.
(ii) Solve the differential equation

$$
\begin{equation*}
\frac{\mathrm{d} x}{\mathrm{~d} t}=-\sin x \tan x \cot t \tag{5}
\end{equation*}
$$

given that $x=\frac{1}{6} \pi$ when $t=\frac{1}{2} \pi$.

8 (i) Given that $\frac{2 t}{(t+1)^{2}}$ can be expressed in the form $\frac{A}{t+1}+\frac{B}{(t+1)^{2}}$, find the values of the constants $A$ and $B$.
(ii) Show that the substitution $t=\sqrt{2 x-1}$ transforms $\int \frac{1}{x+\sqrt{2 x-1}} \mathrm{~d} x$ to $\int \frac{2 t}{(t+1)^{2}} \mathrm{~d} t$.
(iii) Hence find the exact value of $\int_{1}^{5} \frac{1}{x+\sqrt{2 x-1}} \mathrm{~d} x$.

9 The parametric equations of a curve are

$$
x=2 \theta+\sin 2 \theta, \quad y=4 \sin \theta
$$

and part of its graph is shown below.

(i) Find the value of $\theta$ at $A$ and the value of $\theta$ at $B$.
(ii) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\sec \theta$.
(iii) At the point $C$ on the curve, the gradient is 2 . Find the coordinates of $C$, giving your answer in an exact form.

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