

5.

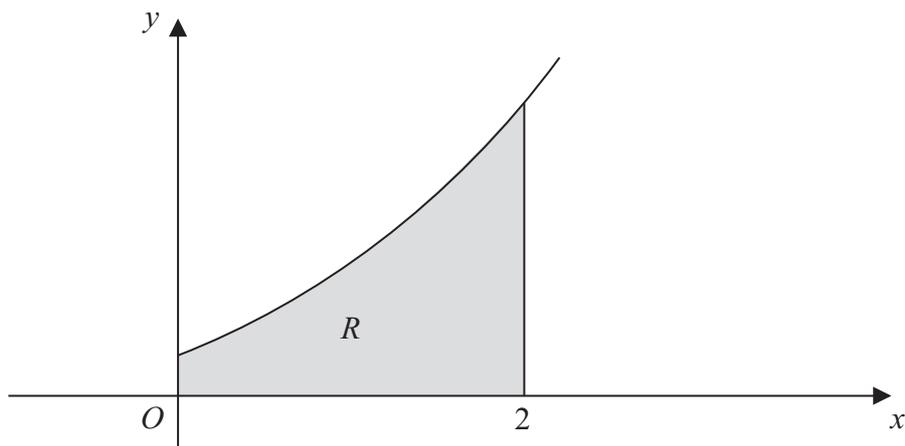


Figure 3

The shaded region R is bounded by the curve with equation $y = (x + 1)^2$, the x -axis, the y -axis and the line with equation $x = 2$, as shown in Figure 3. The region R is rotated through 2π radians about the x -axis to form a uniform solid S .

(a) Use algebraic integration to find the x coordinate of the centre of mass of S .

(8)

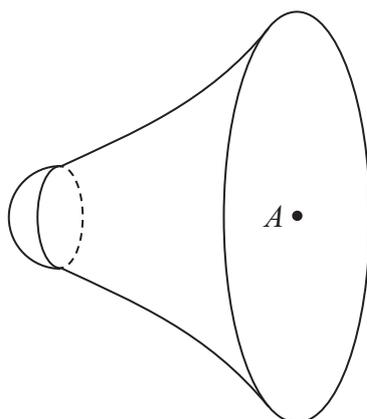


Figure 4

A uniform solid hemisphere is fixed to S to form a solid T . The hemisphere has the same radius as the smaller plane face of S and its plane face coincides with the smaller plane face of S , as shown in Figure 4. The mass per unit volume of the hemisphere is 10 times the mass per unit volume of S . The centre of the circular plane face of T is A . All lengths are measured in centimetres.

(b) Find the distance of the centre of mass of T from A .

(5)



7.

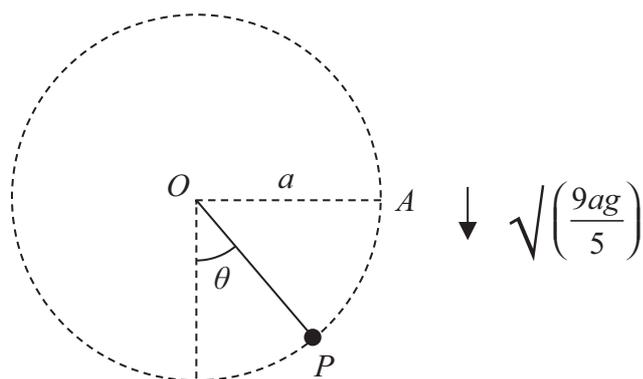


Figure 6

A particle P of mass $5m$ is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held at the point A , where $OA = a$ and OA is horizontal, as shown in Figure 6. The particle is projected vertically downwards with speed $\sqrt{\left(\frac{9ag}{5}\right)}$. When the string makes an angle θ with the downward vertical through O and the string is still taut, the tension in the string is T .

(a) Show that $T = 3mg(5 \cos \theta + 3)$. (6)

At the instant when the particle reaches the point B the string becomes slack.

(b) Find the speed of P at B . (3)

At time $t = 0$, P is at B .

At time t , before the string becomes taut once more, the coordinates of P are (x, y) referred to horizontal and vertical axes with origin O . The x -axis is directed along OA produced and the y -axis is vertically upward.

(c) Find

(i) x in terms of t, a and g ,

(ii) y in terms of t, a and g . (7)



