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Mark Scheme

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1	Attempt use of quotient rule to find derivative Obtain $\frac{2(3x-1)-3(2x+1)}{(3x-1)^2}$ Obtain $-\frac{5}{4}$ for gradient Attempt eqn of straight line with numerical gradient Obtain $5x+4y-11=0$	M1 A1 A1 M1 A1	allow for numerator 'wrong way round'; or attempt use of product rule or equiv or equiv obtained from their $\frac{dy}{dx}$; tangent not normal 5 or similar equiv
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2 (i)	Attempt complete method for finding $\cot \theta$ Obtain $\frac{5}{12}$	M1 A1	rt-angled triangle, identities, calculator, ... 2 or exact equiv
(ii)	Attempt relevant identity for $\cos 2\theta$ State correct identity with correct value(s) substituted Obtain $-\frac{119}{169}$	M1 A1 A1	$\pm 2\cos^2 \theta \pm 1$ or $\pm 1 \pm 2\sin^2 \theta$ or $\pm(\cos^2 \theta - \sin^2 \theta)$ correct answer only earns 3/3
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3 (a)	Sketch reasonable attempt at $y = x^5$ Sketch straight line with negative gradient Indicate in some way single point of intersection	B1 B1 B1	accept non-zero gradient at O but curvature to be correct in first and third quadrants existing at least in (part of) first quadrant dep *B1 *B1
(b)	Obtain correct first iterate Carry out process to find at least 3 iterates in all Obtain at least 1 correct iterate after the first Conclude 2.175 [0 \rightarrow 2.21236 \rightarrow 2.17412 \rightarrow 2.17480 \rightarrow 2.17479; 1 \rightarrow 2.19540 \rightarrow 2.17442 \rightarrow 2.17480 \rightarrow 2.17479; 2 \rightarrow 2.17791 \rightarrow 2.17473 \rightarrow 2.17479 \rightarrow 2.17479; 3 \rightarrow 2.15983 \rightarrow 2.17506 \rightarrow 2.17479 \rightarrow 2.17479]	B1 M1 A1 A1	allow if not part of subsequent iteration allow for recovery after error; showing at least 3 d.p. in iterates answer required to precisely 3 d.p.
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4 (i)	Obtain derivative of form $k(4t+9)^{-\frac{1}{2}}$ Obtain correct $2(4t+9)^{-\frac{1}{2}}$ Obtain derivative of form $ke^{\frac{1}{2}x+1}$ Obtain correct $3e^{\frac{1}{2}x+1}$	M1 A1 M1 A1	any constant k or (unsimplified) equiv any constant k different from 6 4 or equiv
(ii)	<u>Either:</u> Form product of two derivatives Substitute for t and x in product Obtain 39.7 <u>Or:</u> Obtain $k(4t+9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}+1}}$ Obtain correct $6(4t+9)^{-\frac{1}{2}} e^{\frac{1}{2}(4t+9)^{\frac{1}{2}+1}}$ Substitute $t = 4$ to obtain 39.7	M1 M1 A1 M1 A1	numerical or algebraic using $t = 4$ and calculated value of x allow ± 0.1 ; allow greater accuracy differentiating $y = 6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}+1}}$ or equiv allow ± 0.1 ; allow greater accuracy
5 (i)	Obtain $R = \sqrt{17}$ or 4.12 or 4.1 Attempt recognisable process for finding α Obtain $\alpha = 14$	B1 M1 A1	or greater accuracy allow for sin/cos confusion 3 or greater accuracy 14.036...

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- (ii) Attempt to find at least one value of $\theta + \alpha$ M1
 Obtain or imply value 61 A1√ following R value; or value rounding to 61
 Obtain 46.9 A1 allow ± 0.1 ; allow greater accuracy
 Show correct process for obtaining second angle M1
 Obtain -75 A1 5 allow ± 0.1 ; allow greater accuracy; max of 4/5 if extra angles between -180 and 180

- 6 (i) Obtain integral of form $k(3x + 2)^{\frac{1}{2}}$ M1 any constant k
 Obtain correct $\frac{2}{3}(3x + 2)^{\frac{1}{2}}$ A1 or equiv
 Substitute limits 0 and 2 and attempt evaluation M1 for integral of form $k(3x + 2)^n$
 Obtain $\frac{2}{3}(8^{\frac{1}{2}} - 2^{\frac{1}{2}})$ A1 4 or exact equiv suitably simplified

- (ii) State or imply $\pi \int \frac{1}{3x + 2} dx$ or unsimplified version B1 allow if dx absent or wrong
 Obtain integral of form $k \ln(3x + 2)$ M1 any constant k involving π or not
 Obtain $\frac{1}{3}\pi \ln(3x + 2)$ or $\frac{1}{3}\ln(3x + 2)$ A1
 Show correct use of $\ln a - \ln b$ property M1
 Obtain $\frac{1}{3}\pi \ln 4$ A1 5 or (similarly simplified) equiv

- 7 (i) State a in x -direction B1 or clear equiv
 State factor 2 in x -direction B1 2 or clear equiv
- (ii) Show (largely) increasing function crossing x -axis M1 with correct curvature
 Show curve in first and fourth quadrants only A1 2 not touching y -axis and with no maximum point; ignore intercept
- (iii) Show attempt at reflecting negative part in x -axis M1
 Show (more or less) correct graph A1√ 2 following their graph in (ii) and showing correct curvatures
- (iv) Identify $2a$ as asymptote or $2a + 2$ as intercept B1 allow anywhere in question
 State $2a < x \leq 2a + 2$ B1 2 allow $<$ or \leq for each inequality

- 8 (i) Obtain $-2xe^{-x^2}$ as derivative of e^{-x^2} B1
 Attempt product rule *M1 allow if sign errors or no chain rule
 Obtain $8x^7e^{-x^2} - 2x^9e^{-x^2}$ A1 or (unsimplified) equiv
Either: Equate first derivative to zero and attempt solution M1 dep *M; taking at least one step of solution
 Confirm 2 A1 5 AG
Or: Substitute 2 into derivative and show attempt at evaluation M1
 Obtain 0 A1 (5) AG; necessary correct detail required

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<p>(ii) Attempt calculation involving attempts at y values</p> <p>Attempt $k(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$</p> <p>Obtain $\frac{1}{6}(0 + 4 \times 0.00304 + 2 \times 0.36788 + 4 \times 2.70127 + 4.68880)$</p> <p>Obtain 2.707</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>with each of 1, 4, 2 present at least once as coefficients</p> <p>with attempts at five y values corresponding to correct x values</p> <p>or equiv with at least 3 d.p. or exact values</p> <p>4 or greater accuracy; allow ± 0.001</p>
<p>(iii) Attempt $4(y \text{ value}) - 2(\text{part (ii)})$</p> <p>Obtain 13.3</p>	<p>M1</p> <p>A1</p>	<p>or equiv</p> <p>2 or greater accuracy; allow ± 0.1</p>

<p>9 (i) State $-2 \leq y \leq 2$</p> <p>State $y \leq 4$</p>	<p>B1</p> <p>B1</p>	<p>allow $<$; any notation</p> <p>2 allow $<$; any notation</p>
<p>(ii) Show correct process for composition</p> <p>Obtain or imply 0.959 and hence 2.16</p> <p>Obtain $g(0.5) = 3.5$</p> <p>Observe that 3.5 not in domain of f</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>right way round</p> <p>AG; necessary detail required</p> <p>or (unsimplified) equiv</p> <p>4 or equiv</p>
<p>(iii) Relate quadratic expression to at least one end of range of f</p> <p>Obtain both of $4 - 2x^2 < -2$ and $4 - 2x^2 > 2$</p> <p>Obtain at least two of the x values $-\sqrt{3}, -1, 1, \sqrt{3}$</p> <p>Obtain all four of the x values</p> <p>Attempt solution involving four x values</p> <p>Obtain $x < -\sqrt{3}, -1 < x < 1, x > \sqrt{3}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>or equiv</p> <p>or equiv; allow any sign in each ($<$ or \leq or $>$ or \geq or $=$)</p> <p>to produce at least two sets of values</p> <p>6 allow \leq instead of $<$ and/or \geq instead of $>$</p>