



**ADVANCED SUBSIDIARY GCE
MATHEMATICS**

4725/01

Further Pure Mathematics 1

FRIDAY 11 JANUARY 2008

Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

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- 1 The transformation S is a shear with the y -axis invariant (i.e. a shear parallel to the y -axis). It is given that the image of the point $(1, 1)$ is the point $(1, 0)$.
- (i) Draw a diagram showing the image of the unit square under the transformation S . [2]
- (ii) Write down the matrix that represents S . [2]
- 2 Given that $\sum_{r=1}^n (ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b . [5]
- 3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α , β and γ .
- (i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]
- (ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]
- 4 The complex number $3 - 4i$ is denoted by z . Giving your answers in the form $x + iy$, and showing clearly how you obtain them, find
- (i) $2z + 5z^*$, [2]
- (ii) $(z - i)^2$, [3]
- (iii) $\frac{3}{z}$. [3]
- 5 The matrices \mathbf{A} , \mathbf{B} and \mathbf{C} are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 4 \quad -1)$. Find
- (i) $\mathbf{A} - 4\mathbf{B}$, [2]
- (ii) \mathbf{BC} , [4]
- (iii) \mathbf{CA} . [2]
- 6 The loci C_1 and C_2 are given by
- $$|z| = |z - 4i| \quad \text{and} \quad \arg z = \frac{1}{6}\pi$$
- respectively.
- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]
- (ii) Hence find, in the form $x + iy$, the complex number represented by the point of intersection of C_1 and C_2 . [3]

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7 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

(i) Given that \mathbf{A} is singular, find a . [2]

(ii) Given instead that \mathbf{A} is non-singular, find \mathbf{A}^{-1} and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned} \quad [5]$$

8 The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$.

(i) Show that $u_4 = 16$. [2]

(ii) Hence suggest an expression for u_n . [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

9 (i) Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β . Find a quadratic equation with roots α^3 and β^3 . [6]

10 (i) Show that $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} = \frac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n , for

$$\sum_{r=1}^n \frac{3r+4}{r(r+1)(r+2)}. \quad [6]$$

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$, find the value of N . [4]

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