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**General Certificate of Education**

**Mathematics 6360**

**MFP3 Further Pure 3**

**Mark Scheme**

*2009 examination - June series*

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### Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

<b>MFP3</b>				
<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>1(a)</b>	$y(3.1) = y(3) + 0.1\sqrt{3^2 + 2 + 1}$	M1A1	3	Condone > 4dp if correct
	$= 2 + 0.1 \times \sqrt{12} = 2.3464(10..)$	A1		
	$= 2.3464$			
<b>(b)</b>	$y(3.2) = y(3) + 2(0.1)[f(3.1, y(3.1))]$	M1	3	ft on candidate's answer to (a)  CAO Must be 2.720
	$.... = 2 + 2(0.1)[\sqrt{(3.1^2 + 2.3464 + 1)}]$	A1F		
	$.... = 2 + 0.2 \times 3.599499.. = 2.719(89..)$ $= 2.720$	A1		
<b>Total</b>			<b>6</b>	
<b>2</b>	IF is $e^{\int -\tan x \, dx}$ $= e^{\ln(\cos x) + c}$ $= (k) \cos x$ $\cos x \frac{dy}{dx} - y \tan x \cos x = 2 \sin x \cos x$ $\frac{d}{dx}(y \cos x) = 2 \sin x \cos x$ $y \cos x = \int 2 \sin x \cos x \, dx$ $y \cos x = \int \sin 2x \, dx$ $y \cos x = -\frac{1}{2} \cos 2x + c$ $2 = -\frac{1}{2} + c$ $c = \frac{5}{2}$ $y \cos x = -\frac{1}{2} \cos 2x + \frac{5}{2}$	M1 A1 A1F  M1 A1F m1 A1 m1 A1	9	Award even if negative sign missing OE Condone missing $c$ ft earlier sign error  LHS as $\frac{d}{dx}(y \times \text{IF})$ PI ft on $c$ 's IF provided no exp or logs Double angle or substitution OE for integrating $2 \sin x \cos x$ ACF Boundary condition used to find $c$  ACF eg $y \cos x - 2 + \sin^2 x$ Apply ISW after ACF
<b>Total</b>			<b>9</b>	

**MFP3 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>3(a)</b>	Centre of circle is $M(3, 4)$  $A(6, 8)$	B1  B1	  2	PI
<b>(b)(i)</b>	$k = OA = 10$ $\tan \alpha = \frac{y_A}{x_A} = \frac{4}{3}$	B1  B1	  2	SC “ $r = 10$ and $\tan \theta = \frac{8}{6}$ ” = B1 only
<b>(b)(ii)</b>	$x^2 + y^2 - 6x - 8y + 25 = 25$  $r^2 - 6r \cos \theta - 8r \sin \theta = 0$	B1  M1M1	  4	If polar form before expansion award the B1 for correct expansions of both $(r \cos \theta - m)^2$ and $(r \sin \theta - n)^2$ where $(m, n) = (3, 4)$ or $(m, n) = (4, 3)$ 1st M1 for use of any one of $x^2 + y^2 = r^2$ , $x = r \cos \theta$ , $y = r \sin \theta$  2nd M1 for use of these to convert the form $x^2 + y^2 + ax + by = 0$ correctly to the form $r^2 + ar \cos \theta + br \sin \theta = 0$
	$\{r = 0, \text{origin}\}$ Circle: $r = 6 \cos \theta + 8 \sin \theta$  <b>ALTn</b> Circle has eqn $r = OA \cos(\alpha - \theta)$ $r = OA \cos \alpha \cos \theta + OA \sin \alpha \sin \theta$ Circle: $r = 6 \cos \theta + 8 \sin \theta$	A1  (M2) (m1) (A1)		NMS Mark as 4 or 0  OE
	<b>Total</b>		<b>8</b>	

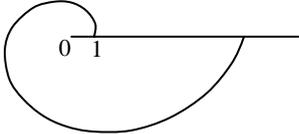
**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
4	$\int \left( \frac{1}{x} - \frac{4}{4x+1} \right) dx = \ln x - \ln(4x+1) \{+c\}$ $I = \lim_{a \rightarrow \infty} \int_1^a \left( \frac{1}{x} - \frac{4}{4x+1} \right) dx$ $= \lim_{a \rightarrow \infty} [\ln x - \ln(4x+1)]_1^a$ $= \lim_{a \rightarrow \infty} \left[ \ln \left( \frac{a}{4a+1} \right) - \ln \frac{1}{5} \right]$ $= \lim_{a \rightarrow \infty} \left[ \ln \left( \frac{1}{4 + \frac{1}{a}} \right) - \ln \frac{1}{5} \right]$ $= \ln \frac{1}{4} - \ln \frac{1}{5} = \ln \frac{5}{4}$	<p>B1</p> <p>M1</p> <p>m1</p> <p>m1</p> <p>A1</p>	5	<p>OE</p> <p><math>\infty</math> replaced by <math>a</math> (OE) and <math>\lim_{a \rightarrow \infty}</math></p> <p><math>\ln a - \ln(4a+1) = \ln \left( \frac{a}{4a+1} \right)</math>  <b>and</b> previous M1 scored</p> <p><math>\ln \left( \frac{a}{4a+1} \right) = \ln \left( \frac{1}{4 + \frac{1}{a}} \right)</math> <b>and</b>                      previous M1m1 scored</p> <p>CSO</p>
<b>Total</b>			<b>5</b>	
5(a)	$-k \sin x + 2k \cos x + 5k \sin x = 8 \sin x + 4 \cos x$	M1 A1 A1	3	Differentiation and subst. into DE
(b)	Auxl eqn $m^2 + 2m + 5 = 0$ $m = \frac{-2 \pm \sqrt{4-20}}{2}$ $m = -1 \pm 2i$ CF: $\{y_c\} = e^{-x}(A \sin 2x + B \cos 2x)$ GS $\{y\} = e^{-x}(A \sin 2x + B \cos 2x) + k \sin x$ When $x = 0, y = 1 \Rightarrow B = 1$ $\frac{dy}{dx} = -e^{-x}(A \sin 2x + B \cos 2x)$ $+ e^{-x}(2A \cos 2x - 2B \sin 2x) + k \cos x$	M1 A1 A1F B1F B1F		Formula or completing sq. PI  ft provided $m$ is not real ft on CF + PI; must have 2 arb consts
	When $x = 0, \frac{dy}{dx} = 4 \Rightarrow 4 = -B + 2A + k$ $\Rightarrow A = \frac{3}{2}$ $y = e^{-x} \left( \frac{3}{2} \sin 2x + \cos 2x \right) + 2 \sin x$	M1 A1	8	Product rule  PI  CSO
<b>Total</b>			<b>11</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
<b>6(a)(i)</b>	$f(x) = (9 + \tan x)^{\frac{1}{2}}$ $\text{so } f'(x) = \frac{1}{2}(9 + \tan x)^{-\frac{1}{2}} \sec^2 x$ $f''(x) = -\frac{1}{4}(9 + \tan x)^{-\frac{3}{2}} \sec^4 x$ $+ \frac{1}{2}(9 + \tan x)^{-\frac{1}{2}} (2 \sec^2 x \tan x)$	M1 A1  M1 A1	4	Chain rule  Product rule, OE ACF
<b>(a)(ii)</b>	$f(0) = 3$ $f'(0) = \frac{1}{2}(9)^{-\frac{1}{2}} = \frac{1}{6};$ $f''(0) = -\frac{1}{4}(9)^{-\frac{3}{2}} = -\frac{1}{108}$ $f(x) \approx f(0) + x f'(0) + \frac{1}{2} x^2 f''(0)$ $(9 + \tan x)^{\frac{1}{2}} \approx 3 + \frac{x}{6} - \frac{x^2}{216}$	B1  M1  A1	3	Both attempted and at least one correct ft on c's $f'(x)$ and $f''(x)$  CSO AG
<b>(b)</b>	$\frac{f(x) - 3}{\sin 3x} \approx \frac{\frac{x}{6} - \frac{x^2}{216} \dots}{3x - \frac{(3x)^3}{3!} \dots}$ $\approx \frac{\frac{1}{6} - \frac{x}{216} \dots}{3 - \dots}$ $\lim_{x \rightarrow 0} \left[ \frac{f(x) - 3}{\sin 3x} \right] = \frac{1}{18}$	M1  m1  A1	3	Using series expns.  Dividing numerator and denominator by $x$ to get constant term in each
<b>Total</b>			<b>10</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
7(a)	$\text{Area} = \frac{1}{2} \int \left(1 + 6e^{-\frac{\theta}{\pi}}\right)^2 d\theta$ $= \frac{1}{2} \int_0^{2\pi} \left(1 + 12e^{-\frac{\theta}{\pi}} + 36e^{-\frac{2\theta}{\pi}}\right) d\theta$ $= \frac{1}{2} \left[ \theta - 12\pi e^{-\frac{\theta}{\pi}} - 18\pi e^{-\frac{2\theta}{\pi}} \right]_0^{2\pi}$ $= \pi (16 - 6e^{-2} - 9e^{-4})$	M1 B1 B1 m1 A1	5	Use of $\frac{1}{2} \int r^2 d\theta$ Correct expansion of $\left(1 + 6e^{-\frac{\theta}{\pi}}\right)^2$ Correct limits Correct integration of at least two of the three terms $1$ , $p e^{-\frac{\theta}{\pi}}$ , $q e^{-\frac{2\theta}{\pi}}$ ACF
(b)	 <p>End-points <math>(1, 0)</math> and <math>(e^2, 2\pi)</math></p>	B1 B1 B2,1,0	4	Going the correct way round the pole Increasing in distance from the pole Correct end-points B1 for each pair or for 1 and $e^2$ shown on graph in correct positions
(c)	$e^{\frac{\theta}{\pi}} = 1 + 6e^{-\frac{\theta}{\pi}}$ $\left(e^{\frac{\theta}{\pi}}\right)^2 - e^{\frac{\theta}{\pi}} - 6 = 0$ $\left(e^{\frac{\theta}{\pi}} - 3\right)\left(e^{\frac{\theta}{\pi}} + 2\right) = 0$ $e^{\frac{\theta}{\pi}} > 0 \text{ so } e^{\frac{\theta}{\pi}} = 3$ <p>Polar coordinates of <math>P</math> are <math>(3, \pi \ln 3)</math></p>	M1 m1 m1 E1 A1	5	Elimination of $r$ or $\theta$ [ $r = 1 + \frac{6}{r}$ ] Forming quadratic in $e^{\frac{\theta}{\pi}}$ or in $e^{-\frac{\theta}{\pi}}$ or in $r$ . [ $r^2 - r - 6 = 0$ ] OE Rejection of negative 'solution' PI [ $r = 3$ ]
<b>Total</b>			<b>14</b>	

**MFP3 (cont)**

Q	Solution	Marks	Total	Comments
<b>8(a)(i)</b>	$\frac{dx}{dt} = 2t$	B1	3	PI or for $\frac{dt}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$
	$\frac{dx}{dt} \frac{dy}{dx} = \frac{dy}{dt}$	M1		OE Chain rule $\frac{dy}{dx} = \dots$ or $\frac{dy}{dt} = \dots$
	$2t \frac{dy}{dx} = \frac{dy}{dt}$ so $2\sqrt{x} \frac{dy}{dx} = \frac{dy}{dt}$	A1		AG
<b>(a)(ii)</b>	$\frac{d}{dx} \left( 2\sqrt{x} \frac{dy}{dx} \right) = \frac{d}{dx} \left( \frac{dy}{dt} \right) = \frac{dt}{dx} \frac{d}{dt} \left( \frac{dy}{dt} \right)$	M1	3	$\frac{d}{dx}(f(t)) = \frac{dt}{dx} \frac{d}{dt}(f(t))$ OE eg $\frac{d}{dt}(g(x)) = \frac{dx}{dt} \frac{d}{dx}(g(x))$
	$2\sqrt{x} \frac{d^2y}{dx^2} + x^{-\frac{1}{2}} \frac{dy}{dx} = \frac{1}{2t} \frac{d^2y}{dt^2}$ $4t\sqrt{x} \frac{d^2y}{dx^2} + 2tx^{-\frac{1}{2}} \frac{dy}{dx} = \frac{d^2y}{dt^2}$ $\Rightarrow 4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = \frac{d^2y}{dt^2}$	M1  A1		Product rule OE  AG Completion
<b>(b)</b>	$4x \frac{d^2y}{dx^2} + 2(1+2\sqrt{x}) \frac{dy}{dx} - 3y = 0$ $(4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx}) + 2(2\sqrt{x} \frac{dy}{dx}) - 3y = 0$ $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 3y = 0$	M1 A1	2	Use of either (a)(i) or (a)(ii) AG Completion
	<b>(c)</b> $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 3y = 0$ (*) Auxl. Eqn. $m^2 + 2m - 3 = 0$ $(m+3)(m-1) = 0$ $m = -3$ and $1$ GS of (*) $\{y\} = Ae^{-3t} + Be^t$ $\Rightarrow y = Ae^{-3\sqrt{x}} + Be^{\sqrt{x}}$	M1 A1 M1 A1		4
<b>Total</b>			<b>12</b>	
<b>TOTAL</b>			<b>75</b>	