Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Withdrawn

1. $$\mathbf{M} = \begin{pmatrix} a & 1 \\ 1 & 2-a \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find det $\mathbf{M}$ in terms of a.

(2)

A triangle T is transformed to T' by the matrix $\mathbf{M}$.

Given that the area of T' is 0,

(b) find the value of a.

(3)

2. $$\mathrm{f}(z) = z^3 + 5z^2 + 11z + 15$$

Given that $z = 2\mathrm{i} - 1$ is a solution of the equation $\mathrm{f}(z) = 0$, use algebra to solve $\mathrm{f}(z) = 0$ completely.

(5)

3. $$z_1 = \frac{1}{2}(1 + \mathrm{i}\sqrt{3}), \quad z_2 = -\sqrt{3} + \mathrm{i}$$

(a) Express z_1 and z_2 in the form $r(\cos\theta + \mathrm{i}\sin\theta)$ giving exact values of r and θ.

(4)

(b) Find $|z_1 z_2|$.

(2)

(c) Show and label z_1 and z_2 on a single Argand diagram.

(2)

4. The hyperbola H has equation

$$xy = 3$$

The point $Q(1, 3)$ is on H.

(a) Find the equation of the normal to H at Q in the form $y = ax + b$, where a and b are constants.

(5)

The normal at Q intersects H again at the point R.

(b) Find the coordinates of R.

(5)

5. Prove, by induction, that $3^{2n} + 7$ is divisible by 8 for all positive integers n.

(6)

6. A curve C is in the form of a parabola with equation $y^2 = 4x$.

$P(p^2, 2p)$ and $Q(q^2, 2q)$ are points on C where $p > q$.

(a) Find an equation of the tangent to C at P.

(5)

(b) The tangent at P and the tangent at Q are perpendicular and intersect at the point $R(-1, 2)$.

(i) Find the exact value of p and the exact value of q.

(4)

(ii) Find the area of the triangle PQR.

(4)

7. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(r-1) = \frac{n(n+1)(3n+2)(n-1)}{12}$$

for all positive integers n.

(5)

(b) Hence find the sum of the series

$$10^2 \times 9 + 11^2 \times 10 + 12^2 \times 11 + \ldots + 50^2 \times 49$$

(3)

8. $$\mathrm{f}(x) = x^3 - 2x - 3$$

(a) Show that $\mathrm{f}(x) = 0$ has a root, α, in the interval $[1, 2]$.

(3)

(b) Starting with the interval $[1, 2]$, use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

(c) Using $x_0 = 1.8$ as a first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(5)

9. With reference to a fixed origin O and coordinate axes Ox and Oy, a transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$$

(a) Find $\mathbf{A}^2$.

(2)

(b) Show that the matrix $\mathbf{A}$ is non-singular.

(2)

(c) Find $\mathbf{A}^{-1}$.

(2)

The transformation represented by matrix $\mathbf{A}$ maps the point P onto the point Q.

Given that Q has coordinates $(k - 1, 2 - k)$, where k is a constant,

(d) show that P lies on the line with equation $y = 4x - 1$

(3)